



SOUTH CAROLINA
STATE DEPARTMENT
OF EDUCATION

South Carolina Department of Education Support for Implementing the Common Core State Standards for Mathematics

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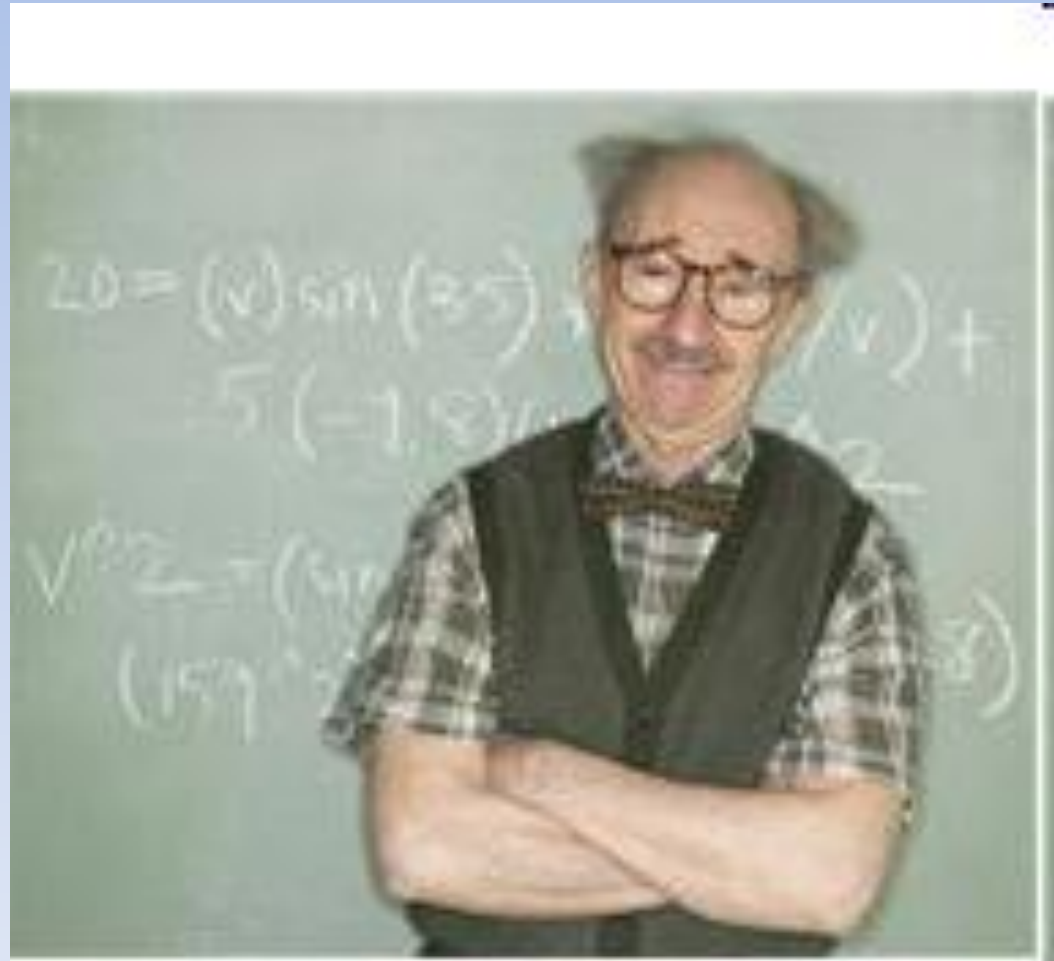
Office of Teacher Effectiveness Content Knowledge Team

**“STEM” - Science,
Technology, Engineering and
Mathematics**

Introductions

Presenters

Table Teams



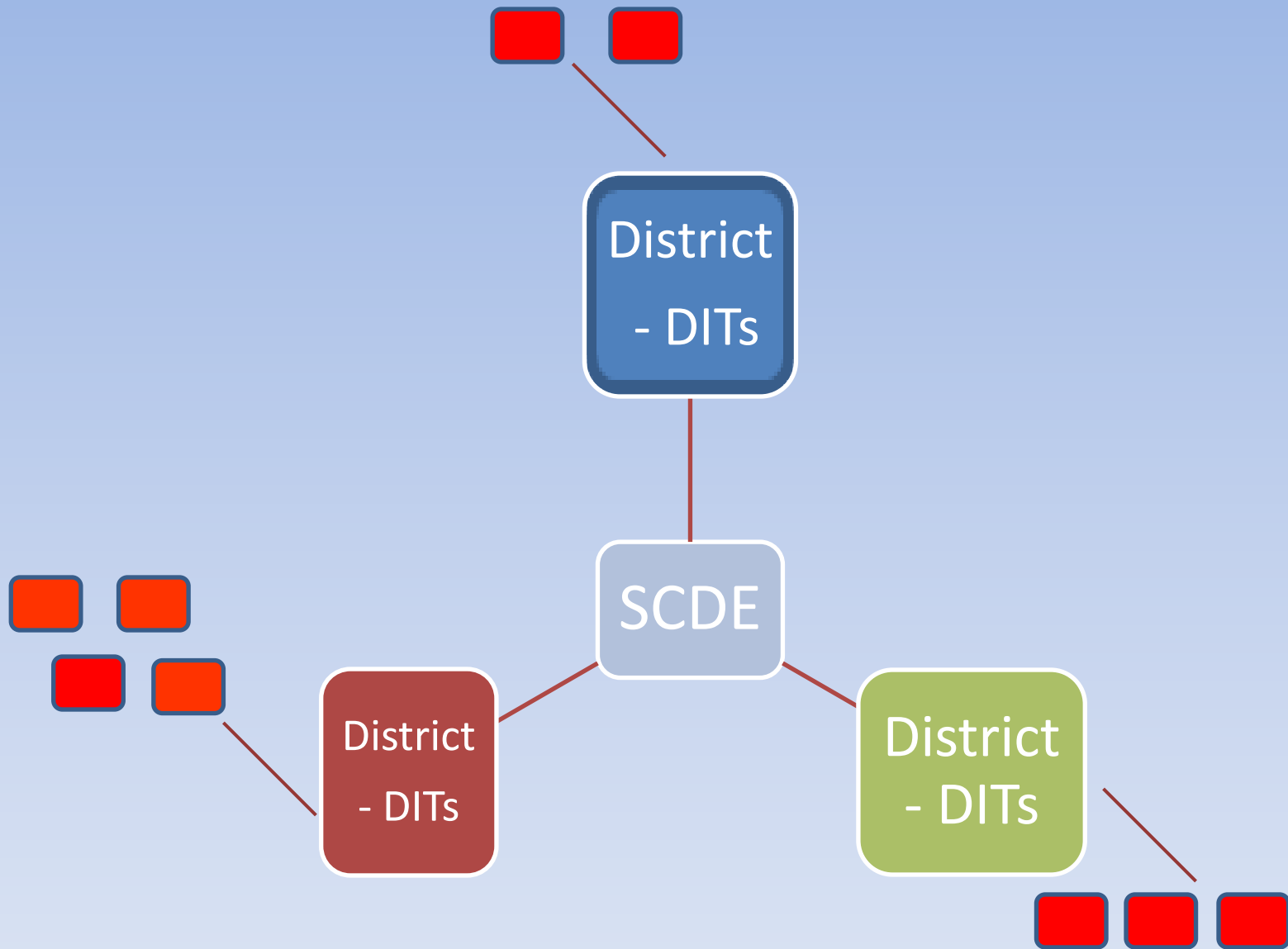
Participants on site are currently introducing themselves in table teams. We will resume the live Web cast in approximately 5 minutes.

You may want to use this time to download materials needed for active participation.

Download/print:

- 1. “Addition Vertical Articulation. . .”**
- 2. Math Practices**
- 3. Tables 1 and 2**





Building Capacity for Implementation

Purpose for Today's Session. . .

- 1. Begin to reflect on how concepts build across grades.**
- 2. Begin to think about the link between the Content Standards and the Mathematical Practices and how the Practices build across grades.**
- 3. Gain insight into expectations of Smarter Balanced**

Common Core State Standards - Addition Vertical Articulation Grades K-8

	1 st Grade	2 nd Grade	3 rd Grade	4 th Grade	5 th Grade	6 th Grade
	<p>Concept - Addition</p> <p><u>Fluently add within 10 1.OA.6</u> Using strategies such as</p> <ul style="list-style-type: none"> Counting on Make ten Decomposing a number leading to a ten The relationship between addition and subtraction Creating easier or known sums <p><u>Add within 20 1.OA.6</u> Using strategies such as</p> <ul style="list-style-type: none"> Counting on Make ten Decomposing a number leading to a ten The relationship between addition and subtraction Creating easier or known sums Applying properties of operations (Associative and Commutative) 1.OA.3 <p>Solve word problems with unknowns in all positions 1.OA.1 (See Glossary Table 1)</p> <ul style="list-style-type: none"> Use 3 or less whole number addends (total 20 or less) Represent the problem using <ul style="list-style-type: none"> Objects Drawings Equations with a symbol for the unknown number <ul style="list-style-type: none"> Determine the unknown number in an addition equation 1.OA.8 <p><u>Relate counting to addition 1.OA.5</u> Counting on 2 to add 2</p>	<p>Concept - Addition</p> <p><u>Fluently add within 20 using mental strategies 2.OA.2</u> Strategies such as</p> <ul style="list-style-type: none"> counting on making ten decomposing a number leading to a ten Using the relationship between addition and subtraction Creating equivalent but easier or known sums <p><u>Write an equation to express an even number as a sum of two equal addends (up to 10) 2.OA.3</u></p> <p><u>By the end of grade 2 know from memory all sums of two 1-digit numbers 2.OA.2</u></p> <p><u>Fluently add within 100 2.NBT.5</u> Add up to four 2-digit numbers 2.NBT.6 Using strategies based on 2.NBT.5 and 2.NBT.6</p> <ul style="list-style-type: none"> Place value Properties of Operations and/or Relationship between addition and subtraction <p>Explain why the strategies work 2.NBT.9</p> <ul style="list-style-type: none"> Explanations may be supported by drawings or objects (footnote 3) <p>Solve <u>one and two-step</u> word problems 2.OA.1</p> <ul style="list-style-type: none"> Unknown in all positions Represent the problem using drawings and equations with a symbol for the unknown number Involving lengths that are 	<p>Concept - Addition - Whole Numbers</p> <p><u>Fluently add within 1000 3.NBT.2</u> Using</p> <ul style="list-style-type: none"> Strategies and algorithms based on place value properties of operations, and/or Relationship between addition and subtraction. Place value understanding to round whole numbers to the nearest 10 or 100 3.NBT.1 <p>Solve two-step word problems using the four operations. 3.OA.8</p> <ul style="list-style-type: none"> Represent the problems using equations with a letter standing for the unknown quantity. Use Order of Operations Assess the reasonableness of answers using mental computation and estimation strategies including rounding Identify arithmetic patterns (including patterns in the addition table or multiplication table), and Explain them using properties of operations. 3.OA.9 	<p>Concept - Addition - Whole Numbers</p> <p><u>Fluently add multi-digit whole numbers 4.NBT.4</u> Use the standard algorithm Solve multistep word problems using the four operations, including problems in which remainders must be interpreted. 4.OA.3.</p> <ul style="list-style-type: none"> Represent the problems using equations with a letter standing for the unknown quantity Assess the reasonableness of answers using mental computation and estimation strategies including rounding. <ul style="list-style-type: none"> Use place value understanding to round multi-digit whole numbers to any place. 4.NBT.3. <p><u>Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. 4.OA.5</u></p> <p>Concept - Addition - Fractions (Addition and subtraction with unlike denominators in general is not a requirement at this grade.) 4.NF.5</p> <p><u>Add mixed numbers with like denominators 4.NF.3c</u> By</p> <ul style="list-style-type: none"> Replacing each mixed number with an equivalent fraction, and/or Using properties of operations and the relationship between addition and subtraction. <p>Understand a fraction a/b with a</p>	<p>Concept - Addition - Whole Numbers</p> <p><i>Note: Addition with whole numbers is not mentioned in the 5th grade standards but will naturally be included in student work. However, it should not be a focus for teaching because based on 4.NBT.4 students should be fluent with addition and subtraction of whole numbers by the end of 4th grade.</i></p> <p>Concept - Addition - Decimals</p> <p><u>Add decimals to hundredths 5.NBT.7</u> Using</p> <ul style="list-style-type: none"> Concrete models Drawings Strategies based on <ul style="list-style-type: none"> Place value Properties of operations and/or The relationship between addition and subtraction Relate the strategy to a written method and explain the reasoning <p>Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols 5.OA.1</p> <ul style="list-style-type: none"> Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them 5.OA.2 <p>Concept - Addition - Fractions</p> <p><u>Add fractions with unlike denominators (including mixed numbers) 5.NF.1</u> Use equivalent fractions as a solving strategy</p>	<p>Concept - Addition - D</p> <p><u>Fluently add, subtract, and divide multi-digit 6.NS.3</u> Use the standard algorithm for each operation</p> <p><i>NOTE: The following standards were selected from the Expressions and Equations Domain. In order to maintain the intent of the standards, more than the concepts may be included.</i></p> <p><u>Use variables to represent numbers and write expressions when solving a real-world mathematical problem. Understand that a variable represents an unknown, or, depending on the problem, a hand, any number in a set.</u></p> <p><u>Write and evaluate numerical expressions involving whole number exponents. 6.EE.1</u> Evaluate expressions with values of their variables using these symbols.</p> <ul style="list-style-type: none"> Include expressions Strategies and algorithms based on place value Perform arithmetic operations, including involving whole-number exponents, in the conventional order there are no parentheses specify a particular <p><u>Solve real-world and mathematical problems involving writing and solving equations of the form $x + p = q$ and</u></p>

Questions to Guide Individual Review



- 1. What consistencies across grades do you notice – both in content and instruction expectations/suggestions?**
- 2. How might those consistencies impact instruction?**
- 3. How does knowing how a concept is addressed in the grade before and after a particular grade support daily instruction and student understanding?**

Teams are now in a work session. We will resume the Web cast in approximately 5 minutes.

If you downloaded the “Addition Vertical Articulation in CCSS” document please review it now to see how what you notice compares to the Views of on-site participants.



Share Out

Avoid Subjective Phrases

The math gets more complicated across grades.

The math gets more complex across grades.

The math gets harder.

Or some form of the above phrases.

Why avoid?

Questions to Guide Table Group Sharing



- 1. What consistencies across grades do you notice – both in content and instruction expectations/suggestions?**
- 2. How might these consistencies impact instruction?**
- 3. How does knowing how a concept is addressed in the grade before and after a particular grade support daily instruction and student understanding?**

Teams are now in a work session. We will resume the Web cast in approximately 15 minutes.

Teams are sharing consistencies, etc. noticed as a result of reviewing the “Addition Vertical Articulation” document. Get ready to compare your thoughts to on-site participants.



Questions to Guide Whole Group Sharing



- 1. What consistencies across grades do you notice – both in content and instruction expectations/suggestions?**
- 2. How might those consistencies impact instruction?**
- 3. How does knowing how a concept is addressed in the grade before and after a particular grade support daily instruction and student understanding?**

Linking Content and the Mathematical Practices. . .

Quickly read the Mathematical Practices you received when you came in this morning.



Teams are now in a work session. We will resume the webinar in approximately 5 minutes.

Participants are individually reviewing the “Mathematical Practices” handout which you downloaded/printed if you are actively participating.



Linking Content and the Mathematical Practices continued. . .

- 1. For your grade band choose and read one terminal objective and the supporting actions/standards.**
- 2. For that terminal objective discuss with your table group how classroom instruction should look through the lens of the mathematical practices.**

Continued on next slide. . .

Linking Content and the Mathematical Practices continued. . .

After discussion record on chart paper

- a. Terminal Objective**
- b. Standard Delineation (#)**
- c. Math Practices Title**
- d. Suggested strategies that exemplify the identified practices and support the terminal objective**

Teams are now in a work session. We will resume the webinar in approximately 20 minutes.

If you are actively participating we encourage you to also select a terminal objective and engage in the activity at your location.



Mathematical Practices Build Across Grades. . .

Just as the content knowledge addressed in early grades lays the foundation for content understanding in succeeding grades, the classroom experiences in early grades lay the foundation for developing a greater level of mathematical proficiency in succeeding grades.



How do early classroom experiences lay the foundation for developing levels of mathematical proficiency?

- 1. “Make sense of problems and persevere in solving them.”**

Re-read this practice on the handout and underline or highlight what you consider to be key points.



Teams are now in a work session. We will resume the webinar in approximately 3 minutes.

If you are actively participating we encourage you to also re-read Practice #1 and underline points that speak to you.



How might practice #1 be exemplified in the early grades and how does that lay the foundation for deepening levels of mathematical proficiency?

- 1. Read the questions on the handout in the center of your table.**
- 2. Mark the questions which are typical of the types you hear in classrooms or see in instructional materials most often.**
- 3. Share with your table group.**

Teams are now in a work session. We will resume the webinar in approximately 5 minutes.

If you are actively participating we encourage you to read “How Do Early Classroom Experiences Lay the Foundation for Mathematical Proficiency?” and mark the questions as on-site participants are doing.



Analyze Questions to Determine Entry Points for Solving

Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?

(Result Unknown)

Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?

(Change Unknown)

Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?

(Start Unknown)

Analyze Questions to Determine Entry Points for Solving

Susan was shopping and saw a \$160 item on sale at 25% off. She wanted to know how much she would save so she asked the clerk 25% of \$160 is what?

(Result Unknown)

Susan was comparing prices and saw an item on sale for \$120. The original price was \$160. The sale price is what percent of the original price?

(Change Unknown)

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
Put Together/ Take Apart ²	Total Unknown	Addend Unknown	Both Addends Unknown ¹
	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$
Compare ³	Difference Unknown	Bigger Unknown	Smaller Unknown
	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

Types of problems we see most often. . .

1. How do the structures /**situations** differ?
2. Where do you think the “unknown” part usually falls in most texts and teaching?
result unknown, change unknown or start unknown
3. What are the implications related to instructional practice?
4. Which type do you think will be most difficult for students and why?

A brown paper lunch bag and a red apple are shown against a white background. The bag is on the left, and the apple is on the right. The word "Lunch" is written in bold black text across the center of the bag.

Lunch

How might practice #1 be exemplified in the early grades and how does that lay the foundation for deepening levels of mathematical proficiency?

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. <i>How many bunnies are on the grass now?</i> $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. <i>How many bunnies hopped over to the first two?</i> $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. <i>How many bunnies were on the grass before?</i> $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. <i>How many apples are on the table now?</i> $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. <i>How many apples did I eat?</i> $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. <i>How many apples were on the table before?</i> $? - 2 = 3$
	Total Unknown	Addend Unknown	Both Addends Unknown'
Put Together/	Three red apples and two green apples are on the table. <i>How many apples are on the table?</i>	Five apples are on the table. Three are red and the rest are green. <i>How many apples are green?</i>	Grandma has five flowers. <i>How many can she put in her red vase and how many in her blue vase?</i>

The common structures we just examined lay the arithmetic problem solving foundation for algebraic problem solving.

Semantic
and
Computational Forms



Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?

$$2 + 3 =$$

Susan was shopping and saw a \$160 item on sale at 25% off. She wanted to know how much she would save so she asked the clerk 25% of \$160 is what?

$$25\% \times 160 = X$$



Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?

$$2 + ? = 5$$

Susan was comparing prices and saw an item on sale for \$120. The original price was \$160. The sale price is what percent of the original price?

$$120 = X\% \times 160$$

$$X\% \text{ of } 160 = 120$$



$$2 + 3 =$$

$$2 + ? = 5$$



- ***Semantic*** form based on the words in the problem
- Used ***arithmetic*** knowledge to solve.

$$25\% \times 160 = X$$

$$.25 \times 160 = X \quad \text{Convert to decimal and multiply}$$

$$40 = X$$

- *Semantic* form based on the words in the problem to write the equation
- Used *arithmetic* knowledge to solve
- May use *computational form* to solve

$$120 = X\% \times 160$$

$$\frac{120}{160} = \frac{X\% \times 160}{160} \quad \text{Use inverse operation to isolate the variable – Identity Element for multi.}$$

$$.40 = X\%$$

$$40 = X$$

- *Semantic* form based on the words in the problem
- *Arithmetic* and *algebraic* knowledge
- Used *computational* form to solve

7.EE.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Transition to CCSS. . .

Why is it important to address the common structures set forth in Table 1?



Why is it important to examine the structures in Table 1 from a K-8 perspective rather than just a K-2, 3-5 or even a K-5 perspective?

How are the structures, semantic and computational forms, and the ideas about arithmetic approaches and algebraic approaches related?

How do today's experiences support implementation?

What will today's process enable others to experience related to CCSS?

- 1. Begin to reflect on how concepts build across grades.**
- 2. Begin to think about the link between the Content Standards and the Mathematical Practices and how the Practices build across grades.**
- 3. Gain insight into expectations of Smarter Balanced**

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SEDL

http://secc.sedl.org/common_core_videos/index.php?action=view&id=739